# Valuation of derivatives with basis risk

## Background

In financial markets, LIBOR or similar domestic indices, which we will refer to as XIBOR, have been used for both discounting cash flows as well as for projecting forward rates, regardless of the tenor of the forward rate to be projected. This meant that a floating rate bond with rates fixed in advance and paid in arrears was worth par at inception, regardless of the fixing tenor;

 (1)

where Pm is the discount factor maturing at the end of period m, the δm the year fraction of the m:th floating period and L(Tm-1,Tm) the projected Xibor rate fixed at time Tn-1 and paid at time Tn. Also, forward rate agreements could easily be replicated by one short and one long deposit. However, in the summer of 2007, non-negligible spreads when swapping between two different XIBOR tenors in the same currency.

For Cross-Currency IRS trades, CIRS, textbook formulas, like Hull (2006) state that the value of a CIRS is dependent on the Libor rates of the two currencies and the spot FX rate. However, in the CIRS market there have been significant negative spreads on the JPY leg when swapping between JPY and USD as Japanese banks, dominating the JPY LIBOR fixing, were perceived to have a lower credit quality than the USD LIBOR reference banks after the Japanese asset bubble burst in the 90’s. Using the textbook formula for pricing could therefore lead to arbitrage.

## Proposed Solution

### Tenor Basis Swaps

The consensus solution, for example presented by Bianchetti (2010) or Fujii et al (2010) is to separate curves used for discounting from curves used for generating forward rates. This means that one has to generate one individual (forward) curve for each tenor and index. To take a concrete example, in JPY, depending on our exposure, we would possibly need to create (forward) curves for 1M, 3M and 6M LIBOR, 1M, 3M and 6M TIBOR as well as JPY OIS (MUTAN).

This leaves us with the issue of choosing a discount curve. Since, by the no arbitrage argument, two identical future cashflows with the same credit risk must display the same present value, we need a unique discount curve for all single currency IRS trades.

In most currencies there’s a standard tenor and index for the floating leg when quoting swap rates. For EUR this is 6M EURIBOR, for USD it’s 3M USD LIBOR etc. Fujii et al suggest that for this benchmark tenor, the pre-crisis assumption in (1) still holds and Xibor swap rates for the standard fixing tenor are appropriate for discounting[[1]](#footnote-2).

For a swap where the floating rate is fixed on this tenor the following relationship holds at the inception of a swap as the PV is assumed to be 0.

 (2)

where CN is the quoted swap rate for maturity N, ∆n is the year fraction of the n:th fixed coupon, In this case Libor is also the discounting rate which gives the following no-arbitrage relationship between discount factors and the Libor floating payment;

 (3)

In a tenor basis swap we’re swapping one floating rate for another referencing the same index but with a spread applied to one of the legs, e.g. 6M JPY LIBOR vs 3M JPY LIBOR + SPREAD.

 (4)

where κN is market spread of a length K tenor swap. In the case of a 3M/6M tenor swap; K=2M. If we substitute (2) into (4) on both sides we get;

 (5)

We use the following approximation

 (6)

which yields

 (7)

### Currency Basis Swaps

For CIRS there’s a liquid market for swapping 3M USD LIBOR or 3M EURIBOR versus 3M XIBOR in a different currency. This means that we can derive discount factors from such market quotes.

To take a concrete example, for a USDJPY CIRS, where bN is the quoted USDJPY CIRS spread for maturity TN , NJPY the notional in JPY and fx is the spot FX rate. Then the PV at the time of trade is given by;



 (8)

which if we assume that (1) still holds for USD and substitute (2) and (7)[[2]](#footnote-3) on the JPY side, we get;

 (9)

from which we can recursively solve for discount factors. CIRS spreads are usually quoted for maturities 1 year and above but we need discount rates for shorter maturities as well. For this we go to the FX Forward market where FX forward rates, or rather FX points, are quoted for a number of maturities. For simplicity we have decided to use USD as the base in all calculations.

First USD discount rates, rUSD are calculated based on the standard USD zero coupon yield curve used for discounting IRS trades. Then we calculate the far cashflow amount in the non-USD currency;

 (10)

Where dm is days to maturity, dv is days to value, p is FX points for maturity m, sf is the swap point factor, usually 10000 or 100 depending on currency. af is the forward cashflow in the non-USD currency. This gives the implied deposit rate in the non-USD currency which can be used for discounting;

 (11)

where Pv is the USD discount factor for the maturity matching the value date.

Even though there are liquid CIRS spreads quoted against EUR, in Swedbank, all discount curves for CIRS trades are calculated using USD as the base currency, thus discounting all USD legs with a flat LIBOR curve. This could potentially lead to differences in counterparty valuations if they use other currencies as the base. However, unless the PV of a CIRS is significantly different from zero, basing all CIRS discounting on USD as the base should yield results not much different from those using a different base. One should also note that the discount function doesn’t change between floating and fixed legs.

### Index Basis Swaps

In Index basis swaps two different indices in the same currency (and usually the same underlying tenor) are swapped against eachother. For example, JPY TIBOR versus JPY LIBOR or AUD BBSW versus AUD LIBOR. Here the same methodology as for tenor basis swaps can be applied.

### Forward Rate Agreements

Forward rate agreements, FRAs, with cash settlement typically have a contract specific discount rate, typically corresponding to the fixing rate. FRA contracts with cash settlement should therefore have the same forward and discount curve.

1. There are suggestions that derivatives covered by collateral agreements should be discounted by a curve closer approximating the risk free rate than Libor, for example USD OIS, EONIA or SONIA. However, we’re leaving that analysis for later. It can be noted though that clearing house LCH.Clearnet has chosen to use OIS curves for discounting in USD, EUR and GBP (http://www.lchclearnet.com/member\_notices/circulars/2010-06-15.asp) [↑](#footnote-ref-2)
2. We have to make the tenor adjustment on the JPY side as standard JPY IRS rates are quoted against 6M LIBOR while CIRS spreads are quoted against 3M LIBOR. [↑](#footnote-ref-3)